

Part-I 第一卷

Q1 題 1 (8 points 8 分) Solution 解:

For resistance R , only half of the area is conducting, as the other half is blocked by medium-2. Let the voltage between the plates be V , then the electric field

先求電阻 R 。介質-2 不導電，所以只有一半的導電板導電。令兩板之間的電壓為 V ，則電場為

$$E = V/d, \text{ (1 point 1 分)}$$

The current density is 電流密度為 $J = \sigma_1 E$, (1 point 1 分)

The current is 電流為 $I = \frac{1}{2} J a^2 = \frac{a^2}{2} \sigma_1 \frac{V}{d}$. (1 point 1 分)

So the resistance is 因此電阻為 $R = \frac{V}{I} = \frac{2d}{\sigma_1 a^2}$. (1 point 1 分)

For capacitance, it can be treated as two capacitors in parallel. The capacitance on the right is 再求電容。總電容可當作是左右兩個電容并聯。右邊的電容為

$$C_1 = \frac{\epsilon_0 \epsilon_1 a^2}{2d}. \text{ (1 point 1 分)}$$

For the left half, let the electric displacement be D_2 which is the same throughout the region.

在左半邊，令電位移為 D_2 ，電位移處處相同。 (1 point 1 分)

The total free charge on the left half is

左半邊的總自由電荷為

$$Q = \frac{a^2}{2} D_2. \text{ (1 point 1 分)}$$

The total voltage between the two plates is

兩導電板之間的總電壓為

$$V = \frac{1}{2} E_1 d + \frac{1}{2} E_2 d = \frac{d}{2\epsilon_0} D_2 \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right). \text{ (1 point 1 分)}$$

So 因此 $C_2 = \frac{\varepsilon_0 a^2}{d} \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2},$

$$C = C_1 + C_2 = \frac{\varepsilon_0 a^2}{d} \left(\frac{\varepsilon_1}{2} + \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \right). \text{ (1 point 1 分)}$$

Q2 題 2 (10 points 10 分) solution 解:

(a) Let the mass of the blackhole be M , then令黑洞的質量為 M , 則

$$\frac{GM}{r^2} = \frac{v^2}{r}, (2 \text{ points } 2 \text{ 分})$$

So 因此 $\sqrt{\frac{GM}{r}} = v$, (1 point 1 分)

$$n = -\frac{1}{2}. (1 \text{ point } 1 \text{ 分})$$

(b) Let the areal mass density be σ , then令質量面密度為 σ , 則

$$\frac{G\pi\sigma r^2}{r^2} = \frac{v^2}{r}, (2 \text{ points } 2 \text{ 分})$$

So 因此 $\sqrt{G\pi\sigma r} = v$, (1 point 1 分)

$$n = \frac{1}{2}. (1 \text{ point } 1 \text{ 分})$$

(c) Anything reasonable is fine. It DOES NOT have to be dark matter.

任何有一定理由的解釋都行, 不一定非暗物質不可。(2 points 2 分)

Q3 題 3 (10 points 10 分) Solution 解:

Method 1: Using conservation of energy

方法-1: 利用能量守恒

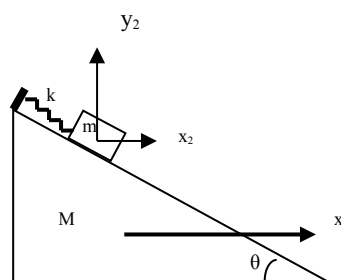
Equations 方程:

$$Mx_1 = -mx_2, \quad y_2 = (x_2 - x_1) \tan \theta = \left(1 + \frac{m}{M}\right)x_2 \tan \theta$$

(1 point 1 分)

All coordinates are in the rest frame. 所有坐標取

自靜止參照系。



The total kinetic energy of the system is

總動能為

$$T = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} m \dot{y}_2^2 \quad (1 \text{ point } 1 \text{ 分})$$

$$= \frac{1}{2} M \frac{m^2}{M^2} \dot{x}_2^2 + \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} m \tan^2 \theta \cdot \dot{x}_2^2 \left(1 + \frac{m}{M}\right)^2 \quad (1 \text{ point } 1 \text{ 分})$$

The total potential energy 總勢能為:

$$V = \frac{1}{2} k [(x_2 - x_1)^2 + y_2^2] - mgy_2 \quad (1 \text{ point } 1 \text{ 分})$$

$$= \frac{1}{2} k \left[\left(1 + \frac{m}{M}\right)^2 x_2^2 + \left(1 + \frac{m}{M}\right)^2 x_2^2 \tan^2 \theta \right] - mg \left(1 + \frac{m}{M}\right) x_2 \tan \theta$$

$$= \frac{1}{2} k x_2^2 \left(1 + \frac{m}{M}\right)^2 \cdot \frac{1}{\cos^2 \theta} - mg \left(1 + \frac{m}{M}\right) x_2 \tan \theta \quad (1 \text{ point } 1 \text{ 分})$$

Using 利用 $ax^2 - bx = a\left(x - \frac{b}{2a}\right)^2 - \frac{b^2}{4a}$, we get 得

$$V = \frac{1}{2} k \left(1 + \frac{m}{M}\right)^2 \frac{1}{\cos^2 \theta} \left(x_2 - \frac{mMg \sin 2\theta}{k(M+m)}\right)^2 - \frac{(mg \sin \theta)^2}{2k} \quad (1 \text{ point } 1 \text{ 分})$$

Make a transfer of coordinate 取坐標變換

$$x \equiv x_2 - \frac{mMg \sin 2\theta}{k(M+m)}, \quad (1 \text{ point } 1 \text{ 分})$$

We reach an expression for the total energy that is of the form of $T + V = ax^2 + b\dot{x}^2 + c$

which must be constant by energy conservation. Let $x = A \cos(\omega t + \phi)$, we get

$$T + V = aA^2 \cos^2(\omega t + \phi) + bA^2 \omega^2 \sin^2(\omega t + \phi) + c = A^2(a - b\omega^2) \cos^2(\omega t + \phi) + bA^2 \omega^2 + c$$

For the total energy to be constant the $\cos^2(\omega t + \phi)$ term must be zero all the time, which

leads to $\omega^2 = a/b$.

我們得到總能量的表達式為 $T + V = ax^2 + b\dot{x}^2 + c$ 。因能量守恒總能量應為常數。令

$x = A \cos(\omega t + \phi)$ ，得

$$T + V = aA^2 \cos^2(\omega t + \phi) + bA^2 \omega^2 \sin^2(\omega t + \phi) + c = A^2(a - b\omega^2) \cos^2(\omega t + \phi) + bA^2 \omega^2 + c$$

作為常數，上式中 $\cos^2(\omega t + \phi)$ 項必須為零。因此 $\omega^2 = a/b$ 。

Therefore, the oscillation frequency ω in this case is

由上式得系統的頻率 ω 為

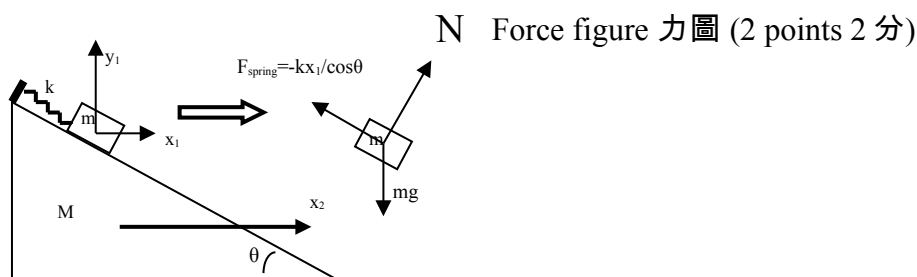
$$\omega^2 = \frac{k(1 + \frac{m}{M})^2 / \cos^2 \theta}{\frac{m}{M^2} + m + m(1 + \frac{m}{M}) \tan^2 \theta} = \frac{k}{m} \frac{1 + \frac{m}{M}}{\cos^2 \theta + (1 + \frac{m}{M}) \sin^2 \theta} = \frac{k}{m} \left(\frac{M + m}{M + m \sin^2 \theta} \right). \quad (1 \text{ point } 1$$

分)

For $\theta = 0$, we have 當 $\theta = 0$ ，得 $\omega^2 = k \left(\frac{1}{m} + \frac{1}{M} \right)$ (1 point 1 分),

For $\theta = 90^\circ$ we have 當 $\theta = 90^\circ$ ，得 $\omega^2 = \frac{k}{m}$. (1 point 1 分)

Method 2: Analytical Mechanics 方法-2：分析力學



Equations 方程

$$m \ddot{x}_1 = -\frac{kx_1}{\cos \theta} - N \sin \theta + m \ddot{x}_2,$$

x_1 is in the frame on the slope. x_1 是相對與斜面的橫坐標。 (1 point 1 分)

$$y_1 = x_1 \tan \theta$$

$$m \ddot{x}_1 \tan \theta = N \cos \theta - mg - kx_1 \tan \theta \quad (1 \text{ point } 1 \text{ 分})$$

$$M \ddot{x}_2 = -(N \sin \theta + kx_1) \quad (1 \text{ point } 1 \text{ 分})$$

Process 解方程過程

Step 1: Eliminate $\ddot{x}_2$ 步驟-1 : 消去 $\ddot{x}_2$

$$m\ddot{x}_1 = -kx_1 - N\sin\theta - \frac{m}{M}(N\sin\theta + kx_1)$$

$$m\ddot{x}_1 \tan\theta = N\cos\theta - mg - kx_1 \tan\theta$$

Step 2: Eliminate N 步驟-2 : 消去 N

$$m\ddot{x}_1 + (1 + \frac{m}{M})kx_1 = -N\sin\theta(1 + \frac{m}{M})$$

$$m\ddot{x}_1 \tan\theta + mg + kx_1 \tan\theta = N\cos\theta$$

Which means 整理後得

$$m\cos\theta\ddot{x}_1 + \cos\theta(1 + \frac{m}{M})kx_1 = -\sin\theta(1 + \frac{m}{M})[m\ddot{x}_1 \tan\theta + mg + kx_1 \tan\theta] \quad (2 \text{ points } 2 \text{ 分})$$

By assuming $x_1 = Ae^{i\omega t}$, the oscillation frequency ω is obtained

設解 $x_1 = Ae^{i\omega t}$, 得頻率 ω

$$\omega^2 = \frac{k(1 + \frac{m}{M})}{m} \frac{\cos\theta + \frac{\sin^2\theta}{\cos\theta}}{\cos\theta + (1 + \frac{m}{M})\frac{\sin^2\theta}{\cos\theta}} = \frac{k}{m} \left(\frac{M+m}{M+m\sin^2\theta} \right). \quad (1 \text{ point } 1 \text{ 分})$$

For $\theta = 0$, we have 當 $\theta = 0$, 得 $\omega^2 = k \left(\frac{1}{m} + \frac{1}{M} \right)$ (1 point 1 分),

For $\theta = 90^\circ$ we have 當 $\theta = 90^\circ$, 得 $\omega^2 = \frac{k}{m}$. (1 point 1 分)

Q4 題 4 (10 points 10 分) Solution 解:

(a) Let the magnetic field be $B(z, t) = B_0 e^{i(\tilde{k}z - \omega t)}$. The k-vector of the wave is $k = \tilde{k}z_0$.

Using the equation $\vec{\nabla} \times E = -\frac{\partial B}{\partial t}$,

令磁場的表達式為 $B(z, t) = B_0 e^{i(\tilde{k}z - \omega t)}$ 。k-矢量為 $k = \tilde{k}z_0$ 。利用方程 $\vec{\nabla} \times E = -\frac{\partial B}{\partial t}$

we get 得 $\vec{B}(z,t) = \frac{k \times E(z,t)}{\omega}$, (1 point 1 分)

$$= \frac{\tilde{k}}{\omega} E_0 (\vec{z}_0 \times \vec{x}_0) e^{i(\tilde{k}z - \omega t)} = \frac{k}{\omega} E_0 y_0 e^{i(kz - \omega t)}$$

So 因此 $\vec{B}_0 = \frac{\tilde{k}}{\omega} E_0 \vec{y}_0$. (1 point 1 分)

(b) Note that 利用 $\tilde{k} = \frac{\omega}{c} \sqrt{\epsilon} + i \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}} \equiv k_R + i k_I$.

So 得 $\langle \vec{S} \rangle = \frac{1}{\mu_0} \langle (\vec{E} \times \vec{B}) \rangle = \frac{1}{2\mu_0} \text{Re}(\vec{E} \times \vec{B}^*)$ (1 point 1 分)

$$= \frac{E_0^2}{2\mu_0 \omega} \vec{z}_0 \text{Re}(k_R - i k_I) e^{-2k_I z} = \frac{1}{2\mu_0} \frac{k_R}{\omega} E_0^2 \vec{z}_0 e^{-2k_I z} = \frac{1}{2\mu_0 c} E_0^2 \vec{z}_0 e^{-2k_I z} \quad (1 \text{ point } 1 \text{ 分})$$

(c) $q = -\frac{d \langle S \rangle}{dz} = \frac{k_I k_R}{\mu_0 \omega} E_0^2 e^{-2k_I z} = \frac{\sigma}{2} \frac{c}{\mu_0 \omega} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}} E_0^2 e^{-2k_I z} = \frac{\sigma}{2} E_0^2 e^{-2k_I z}$. (2 points 2 分)

(d) Joule Heat 焦爾熱 $= \langle \vec{J} \cdot \vec{E} \rangle = \frac{1}{2} \text{Re}(\vec{J} \cdot \vec{E}^*) = \frac{\sigma}{2} \text{Re}(\vec{E} \cdot \vec{E}^*) = \frac{\sigma}{2} E_0^2 e^{-2k_I z}$ (2 points 2 分)

(e) The energy loss of EM wave is equal to the Joule Heat.

電磁波能量的損失等于焦爾熱。 (2 points 2 分)

Q5 (12 points) 題 5 (12 分) Solution 解 :

(a) First law 熱力學第一定律: $dU = -PdV + \delta Q$.

The equation of adiabatic processes is 絕熱過程的方程式為

$$\delta Q = dU + PdV = d(3PV) + PdV = 4PdV + 3VdP = 0 \quad (1 \text{ point } 1 \text{ 分})$$

$$\Rightarrow PV^{4/3} = \text{Constant}$$

$$\Rightarrow PV^{4/3} = \text{常數} \quad (1 \text{ point } 1 \text{ 分})$$

(b) In the Carnot cycle 在卡諾循環過程中:

$$(P_1, V_1) \xrightarrow{\text{Isothermal}} (P_1, V_2) \xrightarrow{\text{Adiabatic}} (P_2, V_3) \xrightarrow{\text{Isothermal}} (P_2, V_4) \xrightarrow{\text{Adiabatic}} (P_1, V_1).$$

Isothermal = 等溫 ; Adiabatic = 絕熱

The heat supplied to the gas during the first isothermal process is

第一個等溫過程中吸收的熱量為

$$Q_1 = 3P_1V_2 - 3P_1V_1 + \int_{V_1}^{V_2} P_1 dV = 4P_1(V_2 - V_1). \quad (1 \text{ point } 1 \text{ 分})$$

(c) Similarly, the heat supplied to the gas during the second isothermal process is

第二個等溫過程中吸收的熱量為

$$Q_2 = 4P_2(V_4 - V_3). \quad (1 \text{ point } 1 \text{ 分})$$

(d) From (a) we have 由(a)得:
$$\begin{aligned} P_1V_2^{4/3} &= P_2V_3^{4/3} \\ P_2V_4^{4/3} &= P_1V_1^{4/3} \end{aligned} \quad (1 \text{ point } 1 \text{ 分})$$

By definition 由定義

$$\frac{T_1}{T_2} = -\frac{Q_1}{Q_2} = -\frac{P_1(V_2 - V_1)}{P_2(V_4 - V_3)} = -\frac{P_1^{1/4}P_2^{3/4}(V_3 - V_4)}{P_2(V_4 - V_3)} = \frac{P_1^{1/4}}{P_2^{1/4}}. \quad (2 \text{ points } 2 \text{ 分})$$

Therefore, one may define the absolute temperature by $T = AP^{1/4}$, where A is an arbitrary constant. Since $T = 1$ when $P = 1$, $T = P^{1/4}$.

因此我們可以定義溫度為 $T = AP^{1/4}$, 其中 A 為任意常數。由 $P = 1$ 時 $T = 1$, 得 $T = P^{1/4}$ 。(1 point 1 分)

(e) The internal energy is then 內能為 $U = 3T^4V$. (1 point 1 分)

Hence the heat capacity is 因此熱容量為 $C_V = \left(\frac{\partial U}{\partial T} \right)_V = 12T^3V$. (1 point 1 分)

(f) The entropy is 熵為 $S = \int_0^T C_V \frac{dT}{T} = 12V \int_0^T T^2 dT = 4T^3V = 4P^{3/4}V$. (2 points 2 分)

Part-II 第二卷

Q1 (16 points) 題 1(16 分)Solution 解 :

- (a) Consider a thin layer of gas of unit area and thickness dr . The pressure at r should be larger than the pressure at $r + dr$ in order to balance the gravity $\frac{GMmn}{r^2}$. So we have $\frac{dP}{dr} = -\frac{GM_s mn}{r^2}$, where n is the molecular number density of the gas.

考慮離太陽 r 處一厚度為 dr 的單位面積氣體。在 r 處的氣壓應比在 $r + dr$ 處的大一點，從而平衡太陽的引力 $\frac{GMmn}{r^2} dr$ 。因此有 $\frac{dP}{dr} = -\frac{GM_s mn}{r^2}$ ，其中 n 為氣體的分子數密度。(1 point 1 分)

We also have the ideal gas law $P = nkT_0$. Replace P with n we get $\frac{dn}{n} = -\frac{GM_s m}{kT_0} \frac{dr}{r^2}$.

另有理想氣體方程 $P = nkT_0$ 。將 P 用 n 代入，得 $\frac{dn}{n} = -\frac{GM_s m}{kT_0} \frac{dr}{r^2}$ 。(1 point 1 分)

Finally, $\rho = \rho_0 e^{\alpha/r}$, where $\alpha = \frac{GM_s m}{kT_0}$.

最後得 $\rho = \rho_0 e^{\alpha/r}$ ，其中 $\alpha = \frac{GM_s m}{kT_0}$ 。(1 point 1 分)

- (b) When $r \rightarrow \infty$, $\rho \rightarrow \rho_0$ instead of zero. That means the gas ball is infinitely large, which is unphysical.

當 $r \rightarrow \infty$ ， $\rho \rightarrow \rho_0$ 而不是零。這意味著氣體球是無限大的，不符合實際情況。(2 points 2 分)

- (c) The amount of energy per second through any concentric sphere shells should be constant. That is, $J_0 = 4\pi r^2 I$.

每秒鐘穿過任意一個同心圓殼的能量應為常數。所以 $J_0 = 4\pi r^2 I$ 。(1 point 1 分)

Then 從而得 $\frac{J_0}{4\pi r^2} = I$ 。(2 points 2 分)

(d) $I(r) = -\sigma \frac{dT}{dr}$, so 因此 $\frac{dT}{dr} = -\frac{J_0}{4\pi\sigma r^2}$, (1 point 1 分)

Then 由此得 $T = \frac{J_0}{4\pi\sigma r}$.

The integral constant should be zero, as T should be zero at large distance.

因為在無窮遠處溫度須為零，所以由積分產生的常數必須為零。(2 points 2 分)

(e) Again 重力和壓力平衡, $\frac{dP}{dr} = -\frac{GM_s m n}{r^2}$.

But now $P = nkT(r) = \frac{J_0}{4\pi\sigma r} kn$. Replace n by P in the first equation, we get

$$\frac{dP}{dr} = -\frac{4\pi GM_s m \sigma P}{kJ_0 r},$$

現在 $P = nkT(r) = \frac{J_0}{4\pi\sigma r} kn$ 。將 P 代入 n ，得 $\frac{dP}{dr} = -\frac{4\pi GM_s m \sigma P}{kJ_0 r}$ (1 point 1 分)

which leads to $P = P_0 \left(\frac{r}{r_0} \right)^{-\beta}$, where $\beta = \frac{4\pi GM_s m \sigma}{kJ_0}$.

從而得 $P = P_0 \left(\frac{r}{r_0} \right)^{-\beta}$ ，其中 $\beta = \frac{4\pi GM_s m \sigma}{kJ_0}$ 。(1 point 1 分)

$$\rho = \frac{4\pi\sigma m r P_0}{kJ_0} \left(\frac{r}{r_0} \right)^{-\beta}. \text{ (1 point 1 分)}$$

This time P and ρ go to zero at large r . 現在的 P 和 ρ 在無窮遠處為零。

(f) From the surface temperatures of the planets we know today we estimate that r_0 is about the radius of the orbit of Mars.

由現在各行星的表面溫度我們可以推測大概和火星的軌道相近。(2 points 2 分)

Q2 (16 points) 題 2 (16 分) Solution 解 :

(a) The number of electrons crossing the junction per second is I/e .

每秒鐘通過界面的電子數為 I/e 。(1 point 1 分)

On average, there are $(\alpha - 0.5)I/e$ electrons flip their spins.

平均有 $(\alpha - 0.5)I / e$ 的電子的自旋反轉。(1 point 1 分)

The net angular momentum change per second is then $(\alpha - 0.5) \frac{Ih}{4\pi e} \times 2$.

每秒鐘角動量的淨變化為 $(\alpha - 0.5) \frac{Ih}{4\pi e} \times 2$ 。(1 point 1 分)

This is equal to the torque so $\tau = (\alpha - 0.5) \frac{h}{2\pi e} I$.

角動量的淨變化等于力矩 $\tau = (\alpha - 0.5) \frac{h}{2\pi e} I$ 。(1 point 1 分)

(b) The equation of motion is $J \frac{d^2\theta}{dt^2} = -\eta \frac{d\theta}{dt} - \kappa\theta$.

導線扭擺的運動方程為 $J \frac{d^2\theta}{dt^2} = -\eta \frac{d\theta}{dt} - \kappa\theta$ 。(1 point 1 分)

Let $\theta(t) = \theta_0 e^{i\tilde{\omega}t}$, where $\tilde{\omega}$ is complex,

令 $\theta(t) = \theta_0 e^{i\tilde{\omega}t}$, 其中 $\tilde{\omega}$ 是複數, we get 得(1 point 1 分)

$\tilde{\omega}^2 - i\gamma\omega - \omega_0^2 = 0$, where 其中 $\omega_0^2 \equiv \kappa / J$, $\gamma \equiv \eta / J$ 。(1 point 1 分)

Let $\tilde{\omega} = \omega_R + i\omega_I$ and solving the equation, we get $\theta = \theta_0 e^{-\omega_I t} e^{i\omega_R t}$,

令 $\tilde{\omega} = \omega_R + i\omega_I$, 并解上述方程, 得 $\theta = \theta_0 e^{-\omega_I t} e^{i\omega_R t}$, (1 point 1 分)

Where 其中 $\omega_I = \gamma / 2$, (1 point 1 分), $\omega_R = \sqrt{\omega_0^2 + \gamma^2 / 4}$ 。(1 point 1 分)

(c) This is a forced oscillation with the force given by $\tau(t) = (\alpha - 0.5) \frac{h}{2e} I_0 e^{i\omega t} = \tau_0 e^{i\omega t}$.

這是個受迫振動問題, 驅使力矩為 $\tau(t) = (\alpha - 0.5) \frac{h}{2e} I_0 e^{i\omega t} = \tau_0 e^{i\omega t}$ 。(1 point 1 分)

The equation of motion is 運動方程為 $J \frac{d^2\theta}{dt^2} = -\eta \frac{d\theta}{dt} - \kappa\theta + \tau(t)$ 。(1 point 1 分)

Let 令 $\theta(t) = \theta_0 e^{i\omega t}$, (1 point 1 分)

we get the oscillation amplitude $\theta_0 = \frac{\tau_0 / J}{\omega_0^2 - \omega^2 + i\gamma\omega}$.

得振動幅度為 $\theta_0 = \frac{\tau_0 / J}{\omega_0^2 - \omega^2 + i\gamma\omega}$ 。(1 point 1 分)

The speed of the side wing is 邊翼的速度為 $v(t) = i\omega d\theta_0 e^{i\omega t}$, (1 point 1 分)

and the electromotive potential is 電動勢為 $\xi(t) = BLv(t) = i\omega dBL\theta_0 e^{i\omega t}$. (1 point 1 分)

Q3 (18 points) 題 3(18 分)Solution 解 :

- (a) The magnetic dipole experiences a torque $\tau = m \times B_0$ which is always perpendicular to the $S \sim z_0$ plane. The torque will turn the direction of S so S rotates around B_0 at constant angular speed.

磁偶極子受到的力矩為 $\tau = m \times B_0$, 其方向始終與 $S \sim z_0$ 平面垂直。力矩改變 S 的方向, 因此 S 繞著 B_0 以勻角速度旋轉。(1 point 1 分)

Let the angle between S and z_0 be θ . The torque is $mB_0 \sin \theta = \mu SB_0 \sin \theta$, while the change of angular momentum over time δt is $\delta S = S \sin \theta \delta \phi$.

令 S 與 z_0 之間的夾角為 θ 。則力矩的大小為 $mB_0 \sin \theta = \mu SB_0 \sin \theta$, 而角動量的變化為 $\delta S = S \sin \theta \delta \phi$ 。(1 point 1 分)

Since 既然 $S \sin \theta \delta \phi = \delta S = \mu SB_0 \sin \theta \delta t$ (1 point 1 分)

We have 我們得 $\omega_0 = \frac{\delta \phi}{\delta t} = \mu B_0$. (1 point 1 分)

- (b) In the reference frame rotating at angular velocity $\omega_0 z_0 = -\mu B_0 z_0$, the spin appears stationary.

在以角速度 $\omega_0 z_0 = -\mu B_0 z_0$ 旋轉的參照系裏, 自旋是不動的。(2 points 2 分)

- (c) The effective B-field is 有效磁場為 $\vec{B}_\omega = -\frac{\omega}{\mu}$. (2 points 2 分)

- (d) In the rotating frame of $-\omega_1 z_0$, B_1 also appears static.

在以角速度 $-\omega_1 z_0$ 旋轉的參照系裏, B_1 是不動的。(1 point 1 分)

Let it be along the X' axis in the rotating frame, the total B-field is

$$\vec{B} = (B_0 - \frac{\omega_1}{\mu})\vec{z}'_0 + B_1\vec{x}'_0$$

令 B_1 在旋轉參照系裏沿 X' 方向，則總磁場為 $\vec{B} = (B_0 - \frac{\omega_1}{\mu})\vec{z}_0' + B_1\vec{x}_0'$ 。 (2 points 2 分)

- (e) In this case, only B_1x_0' remains. The spin will rotate around the x_0' axis at angular speed

$$\omega_1 = \mu B_1,$$

這時的磁場只剩下 B_1x_0' 。自旋繞其以角速度 $\omega_1 = \mu B_1$ 旋轉， (2 points 2 分)

and the time to flip the spin is 倒轉自旋所需的時間為 $t = \frac{1}{2} \frac{2\pi}{\omega_1} = \frac{\pi}{\mu B_1}$ 。 (2 points 2 分)

- (f) In this case the spin will rotate around the total B-field given by (c) at angular speed

$$\omega = \mu \sqrt{\left(B_0 - \frac{\omega}{\mu}\right)^2 + B_1^2}.$$

這時的磁場由(c)給出。自旋的角速度為 $\omega = \mu \sqrt{\left(B_0 - \frac{\omega}{\mu}\right)^2 + B_1^2}$ (3 points 3 分)

~~~~~ End 完 ~~~~~